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Dynamic Determination of the Compressibility of Metals*

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Equation of state data for Duralumin in the pressure range from 0.1 to 0.3 megabar have been determined dynamically by measuring shock and free surface velocity electrically in a plate of 24 ST Duralumin that has been stressed by a high explosive detonation. A theory is presented which allows comparison with data obtained by other experimenters, and which yields the relationship between pressure and compression either at constant entropy or constant temperature. The empirical form chosen for the equation of state ($p=\alpha\mu+\beta\mu^2$) expresses the pressure as a quadratic function of the compression. Experimental techniques are described in detail. Five points are given for the equation of state of Duralumin in the pressure range from approximately 0.15 megabar to 0.33 megabars. Some data are also presented for cadmium and steel.

INTRODUCTION

EARLY in 1945 experimental work was initiated at the Los Alamos Scientific Laboratory for the study of physical properties of materials in the pressure range from 0.1 to 0.3 megabar (10^{12} dynes/cm^2). One-tenth of a megabar had been obtained statically at the Geophysical Laboratory of the Carnegie Institution, but determinations of the equation of state at this pressure become impractical partly because of the elaborate and cumbersome apparatus required and partly because static measurements are subject to uncertainties resulting from creep distortions. An alternative technique, and one particularly suited to the data here desired, is to make the measurements dynamically, thereby eliminating the need for elaborate high pressure equipment with its inherent creep uncertainties.

It is, of course, well known that extreme, though temporary, pressures can be developed by high explosives. With suitably designed apparatus, detonation of a high explosive may be made to produce in the material being investigated a plane shock wave approximately flat-topped in the sense that the pressure in the compressed material is virtually independent of position. The conditions for the existence and stability of such shock waves are discussed in a variety of textbooks.[1]

The theory of propagation of such waves implies that simultaneous determinations of the propagation velocity of the wave and of the mass velocity of the compressed material can be used to infer the equation of state. In addition it is possible under certain conditions to determine shock pressures by means of piezoelectric crystals though this last technique has proved rather difficult to exploit.

THEORY

Detonation of high explosive in contact with a metallic specimen produces pressures in the metal under what are usually termed "Rankine-Hugoniot Shock" conditions. Such conditions will be denoted by appending the subscript D to the symbols appearing in Table I. The mechanical considerations of conservation of mass and conservation of momentum, respectively, lead to the equations:

$$\eta-1=\mu=u/(D-u) \tag{1}$$

$$p_D=\rho_0 Du. \tag{2}$$

The pressures and compressions appearing in Tables II and III were computed by means of Eq. (1) and (2). For many purposes, for example in comparing our data with other data, it is necessary to know the relationship between pressure and compression either at constant entropy or at constant temperature. Such conditions will be specified by the subscripts s and T, respectively. In order to compute the difference between p_D and p_s or

TABLE I. Definitions of symbols and list of units to be used in computation.

Symbol	Definition	Units
p	pressure	megabar$=10^{12}$ dynes/cm^2
v	specific volume	cm^3/g
ρ	density at pressure p	g/cm^3
ρ_0	initial density	g/cm^3
$\eta-1=\mu$	compression-1 $=(\rho-\rho_0)/\rho_0$[a]	.....
e	specific internal energy	cm^3-megabar/g$=10^{12}$ ergs/g
T	absolute temperature	electron volt$=\kappa T=11{,}605$°C
s	specific entropy	cm^3-megabar/g-ev
C_v	specific heat at constant volume$=(\partial e/\partial T)_v$	cm^3-megabar/g-ev
D	shock wave velocity	cm/μsec$=$cm/10^{-6} sec
u	particle or mass velocity	cm/μsec$=$cm/10^{-6} sec
σ	excess of free surface velocity over mass velocity	cm/μsec$=$cm/10^{-6} sec
c	velocity of sound $=(\partial p/\partial\rho)_s^{\frac{1}{2}}$	cm/μsec$=$cm/10^{-6} sec
α_s	isentropic bulk modulus at $p=0$	megabar
β_s	second-order coefficient in empirical isentropic equation of state; see Eq. (17)	megabar

[a] Compression is alternatively defined as $(v_0-v)/v_0=(\eta-1)/\eta$, where v_0 is the normal specific volume.

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[1] e.g., Courant-Friedrichs, *Supersonic Flow and Shock Waves* (Interscience Publishers, Inc., New York, 1948), p. 121 *et seq.*

p_T for some specified compression η, one requires an additional shock condition based on thermodynamic conservation of energy:

$$\Delta e_D = (\tfrac{1}{2}) p_D (v_0 - v), \tag{3}$$

where Δe_D represents the internal energy change required to reach the shock pressure p_D at the final specific volume v. The energy change required to reach this same final volume isentropically may be found by integration:

$$\Delta e_s = \int_v^{v_0} p dv. \tag{4}$$

The energy excess under conditions produced by the shock is thermal energy. This energy difference may be expressed in terms of thermodynamic data by the equation:

$$\Delta e_D - \Delta e_s = \int_{p_s}^{p_D} C_v (\partial T/\partial p)_v dp. \tag{5}$$

Equations (3), (4), and (5) when combined result in an equation from which the difference $p_D - p_s$ may be found. Estimates of the integrand in Eq. (5) are available, and for our immediate purposes, appropriate mean values can be selected with sufficient accuracy. In this case an explicit solution for $(p_D - p_s)$ may be obtained in the simple form:

$$p_D - p_s = \frac{(\tfrac{1}{2}) p_s (v_0 - v) - \int_v^{v_0} p dv}{C_v (\partial T/\partial p)_v - (\tfrac{1}{2})(v_0 - v)}. \tag{6}$$

In order to solve Eq. (6) a preliminary estimate is made for p_s as a function of v at constant entropy, in which case the numerator of Eq. (6) may easily be evaluated. The specified correction may then be applied to the observed shock pressure p_D and a second approximation made for the isentropic pressure-volume relation. Further approximations can readily be made if necessary.

The initial free surface velocity of the target plates with which we have experimented is approximately twice the mass velocity. More precisely, however, the excess σ of the free surface velocity over mass velocity is given by the Riemann velocity

$$\sigma = \int_{\rho D}^{\rho_1} (c/\rho) d\rho \tag{7}$$

an expression which may be derived on the assumption that the material compressed by the shock expands isentropically to density ρ_1 when traversed by a wave of rarefaction. The difficulty involved in evaluating σ stems from the fact that the entropy of the expanding material, though constant, is different from the original entropy, and accordingly, to determine the isentropic equation of state, the experimental data require correction. Before considering the corrections necessary it is convenient to transform Eq. (7) to the form

$$\sigma = \int_0^{p_D} (-\partial v/\partial p)_s^{\frac{1}{2}} dp. \tag{8}$$

Evaluation of σ as specified in Eq. (8) may be facilitated by the following artifice. First eliminate the shock velocity D between Eqs. (1) and (2) obtaining an expression for the mass velocity u in the form

$$u = \int_0^{p_D} \left(\frac{v_0 - v}{p_D}\right)^{\frac{1}{2}} dp, \tag{9}$$

the integrand of which is constant once a particular volume v corresponding to the extreme pressure has been chosen. Combining Eqs. (8) and (9) there results

$$(\sigma - u)/u = (1/p_D) \int_v^{v_1} \left\{1 - \left[(\partial v/\partial p)_s \frac{p_D}{v - v_0}\right]^{\frac{1}{2}}\right\} \left(\frac{\partial p}{\partial v}\right)_s dv, \tag{10}$$

where v_1 is the final specific volume after the isentropic expansion. Equation (10) clearly implies that for sufficiently weak shock the free surface velocity approaches twice the mass velocity. Furthermore, approximate values of the integrand of Eq. (10) suffice to provide an estimate of the difference between u and σ.

In order to determine $(\partial p/\partial v)_s$ at the entropy of the shocked material, one may compute the entropy change Δs due to the shock, then estimate from available thermodynamic data the magnitude of $\partial^2 p/\partial s \partial v$, and thus obtain a corrected value of the desired derivative in the form

$$(\partial p/\partial v)_s = (\partial p/\partial v)_{s0} + (\partial^2 p/\partial s \partial v) \Delta s. \tag{11}$$

A formula for $\partial^2 p/\partial s \partial v$ in terms of readily available thermodynamic data is

$$\frac{\partial^2 p}{\partial s \partial v} = -\frac{T}{C_v^2}\left(\frac{\partial p}{\partial T}\right)_v^2 + \frac{T}{C_v}\frac{\partial^2 p}{\partial T \partial v} + \frac{T^2}{C_v^3}\left(\frac{\partial p}{\partial T}\right)_v^2 \left(\frac{\partial C_v}{\partial T}\right)_v - \frac{2T^2}{C_v^2}\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial^2 p}{\partial T^2}\right)_v \tag{12}$$

in which the first term on the right is the most important.

The entropy excess Δs of the shocked material may be computed by integrating with respect to temperature at the final known volume of the compressed material.

The first step in this process is to determine the isentropic temperature T_s at the compressed volume v from the formula

$$\ln(T_s/T_0) = \int_v^{v_0} (1/C_v)(\partial p/\partial T)_v dv. \tag{13}$$

Next one determines the shock temperature T_D at this same volume by means of the relation

$$T_D - T_s = \frac{\Delta e_D - \Delta e_s}{C_v}. \tag{14}$$

The numerator of Eq. (14) may be evaluated by means of Eqs. (3) and (4) which, with the help of Eq. (6), may be put in the form

$$\Delta e_D - \Delta e_s = \frac{(\tfrac{1}{2}) p_s (v_0 - v) - \int_v^{v_0} p\,dv}{1 - \frac{v_0 - v}{2C_v}\left(\frac{\partial p}{\partial T}\right)_v}. \tag{15}$$

Once T_D and T_s are known, it is very simple to compute Δs, for

$$\Delta s = C_v \ln(T_D/T_s). \tag{16}$$

It should perhaps be noted that Eqs. (14) and (16) are both based on processes occurring at constant volume, in which of course no mechanical work is performed by the system. On the other hand, the integrals in Eqs. (13) and (15) are to be evaluated at constant entropy.

It will be clear that all the corrections to be made depend on information about the temperature and volume dependence of the various thermodynamic variables. Fortunately C_v and $(\partial p/\partial T)_v$ do not vary much under the conditions covered by the observations. The calculations for Duralumin have included an estimate of this variation, though it has been found that the final results would not be significantly changed if both quantities were taken as constants.

The necessary calculations are quite straightforward once an approximate expression for p as a function of compression at constant entropy is known. The empirical form chosen is

$$p_s = \alpha_s \mu + \beta_s \mu^2, \tag{17}$$

the entropy being constant. The value of the constant α_s is inferred from known values of the velocity of sound and of the density under standard laboratory conditions. Thus the data derived from shock measurements are used merely to evaluate the constant β_s. Some question naturally arises as to how α_s is related to the observed sound velocity. Equation (17) is intended to apply to material under such great hydrostatic pressure that any shearing stress is completely negligible both in its magnitude and in its effect on the compression η. Accordingly, it would seem natural to evaluate α_s for conditions under which compression occurs without appreciable shearing stress. But determinations of sound velocity in general are made either with bars, for which $c_1 = (E/\rho)^{\frac{1}{2}}$ where E is Young's modulus, or for large masses of material for which $c_2 = [(k + 4G/3)/\rho]^{\frac{1}{2}}$ where k is the bulk modulus and G the shear modulus. It is the isentropic bulk modulus which relates pressure to compression when the shearing stress is negligible, and accordingly we have assumed that

$$\alpha_s = (\partial p/\partial \mu)_s = -v_0 (\partial p/\partial v)_s = k_s. \tag{18}$$

A value of α_s may be quickly deduced from c_1, if Poisson's ratio ν is known, or from c_2 if the velocity of shear waves $c_3 = (G/\rho)^{\frac{1}{2}}$ is known. In the latter case, the computation is obvious; in the former,

$$\alpha_s = E_s/3(1 - 2\nu). \tag{19}$$

Most of the currently available data on compressibilities at extreme pressures have been obtained isothermally. These data may likewise be fitted well by an equation similar to Eq. (17):

$$p_T = \alpha_T \mu + \beta_T \mu^2. \tag{20}$$

The relation between α_s and α_T is well known to be

$$\alpha_s - \alpha_T = (v_0 T/C_v)(\partial p/\partial T)_v^2. \tag{21}$$

The corresponding difference between β_s and β_T is not so well known, but may be written

$$\beta_s - \beta_T = -(\alpha_s - \alpha_T)\{1 - (\tfrac{1}{2}) v_0 [(1/C_v)(\partial p/\partial T)_v + 3(\partial^2 p/\partial v \partial T)/(\partial p/\partial T)_v - 3(T/C_v)(\partial^2 p/\partial T^2)_v - (T/C_v^2)(\partial p/\partial T)_v (\partial C_v/\partial T)_v]\}. \tag{22}$$

In the course of developing experimental techniques with a view to determining what ultimate precision is possible, it was found convenient to use an alloy of aluminum with superior mechanical properties rather than the pure element for which static compressibilities are available. In order to compare the present work with that of others,[2] it is desirable to estimate the effects of the alloying constituents. This can be done easily if one assumes that the volume of the alloy is equal to the sum of the volumes of its constituents. For many alloys this assumption leads to an excellent estimate of the normal density, and it seems reasonable to expect that its validity is not appreciably worse at high pressures. The subscripts (1) and (2) will be used to denote properties of the constituents; absence of either of these denotes a property of the alloy. The additional subscript (o) refers to a property under standard laboratory conditions. Thus the equations of state involved would be

$$\begin{aligned} p &= \alpha \mu + \beta \mu^2, \\ p &= \alpha_1 \mu_1 + \beta_1 \mu_1^2, \\ p &= \alpha_2 \mu_2 + \beta_2 \mu_2^2, \end{aligned} \tag{23}$$

while the equation connecting the various compressions is

$$\frac{1}{\rho_0(\mu + 1)} = \frac{X_1}{\rho_{01}(\mu_1 + 1)} + \frac{X_2}{\rho_{02}(\mu_2 + 1)}. \tag{24}$$

X_1 and X_2 denote the fractions by mass of the respective constituents. Values of α and β in terms of α_1, α_2, β_1, and

[2] P. W. Bridgman, Proc. Am. Acad. Arts Sci. **77**, 189 (1949).

β_2 may be approximated at small compressions by implicit differentiation. This approximation probably does not give very good average values over the extended range of compressions involved, but it should at least suffice to indicate whether or not observed discrepancies are of the sort to be expected because of variation in composition. The required relations are

$$1/\rho_0\alpha=(X_1/\rho_0\alpha_1)+(X_2/\rho_{02}\alpha_2), \tag{25}$$

and

$$\frac{\beta}{\rho_0\alpha^3}=\frac{X_1}{\rho_{01}\alpha_1^2}\left[\frac{\beta_1}{\alpha_1}+1-\frac{\rho_0X_1}{\rho_{01}}\right]+\frac{X_2}{\rho_{02}\alpha_2^2}\left[\frac{\beta_2}{\alpha_2}+1-\frac{\rho_0X_2}{\rho_{02}}\right]. \tag{26}$$

EXPERIMENTAL TECHNIQUES

The essential details of the experimental technique for the necessary velocity determinations may be visualized by reference to Fig. 1. The material to be studied is machined into the form of a plate perhaps 8 inches in diameter and of a thickness governed by considerations to be discussed. A shock wave is induced in this plate by means of a large block of high explosive (H.E.) detonated simultaneously at all points of its upper surface by means of a suitably designed high explosive lens. Detonation of the latter is initiated electrically in the usual way. At the upper surface of the plate, a high pressure pulse is produced, the magnitude of which depends on the type of high explosive used, and the duration of which depends on the size and shape of the high explosive. In order to achieve a condition of fairly constant pressure at the upper surface of the plate for an appreciable length of time the block of H.E. must be large. For although the instantaneous pressure in the detonated H.E. depends primarily on its chemical and physical properties, the pressure generated by the detonation is immediately relieved at the free surfaces by rarefaction waves. These rarefactions limit the time available for the measurements to a few microseconds and also mean that portions of the plate near the sides never receive the full detonation pressure.

Since the block of high explosive is necessarily of finite thickness (usually 3 to 4 inches), the shock wave in the plate resulting from impact by the detonation wave in the explosive is not quite flat-topped. The shock front is followed immediately by a rarefaction from the back surface which results in an exponential decay. This unloading wave, moving through the plate more rapidly than the shock front, is continuously whittling down the peak pressure. In consequence it is necessary to measure the parameters as a function of thickness of the material.

The free surface velocity at the bottom of the plate (Fig. 1) is ascertained by means of externally placed electrical contactors. These contactors, or "pins," shown in Fig. 1, may be arranged to measure either free surface velocity, by spacing them out behind the plate as shown, or compression wave velocity through the plate by insulating and imbedding the pins in holes drilled to various depths. In either case, the oscillograph records the time at which each pin first touches the metallic surface. In order to synchronize the oscillograph, an extra pin is imbedded in the plate at some convenient level, thus providing an electrical pulse for initiation of the sweep.

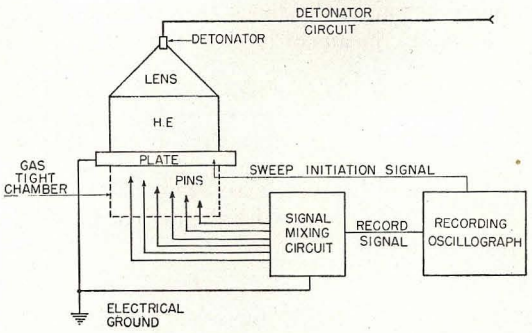

FIG. 1. Apparatus for determining free surface velocity.

Pressure may be measured independently by means of probes made from z-cut tourmaline disks. The calibration constant of the crystals, for the geometry and conditions of these tests, was determined from measurements of u and D in steel. It is advantageous to make these crystals thin. Ours are thicker, about 0.5 mm, than desired but this choice was dictated by practical consideration of existent constructional limitations. Several reverberations are required for equilibrium to be attained between target and crystal and therefore a useful crystal life of about 0.5 μsec is required. If the crystal becomes short circuited before this time a correction factor must be applied for the acoustic mismatch of crystal and specimen plate.

Most of the experimental work on which the present paper is based was performed in 1945. At that time it was realized that elaborate precautions would be required in order to improve the precision of the data. The recent work on Duralumin is inclusive of various improvements in technique which have been discovered over the course of the last five years.

In the first place, if the pins are spaced out behind the plate as shown in Fig. 1, they may be prematurely connected to the plate, and to one another, by ionization of the gas with which they are surrounded. Attempts to insulate the pins from this ionization tend to result in erratic conduction when the metallic surface itself arrives. After much investigation, which included attempted evacuation of the space surrounding the pins, and all sorts of insulations for the pins themselves, it was discovered that little or no preconduction occurs if the gas surrounding the pins is one of the light hydrocarbons, i.e., methane, ethane, propane, or butane. The "gas-tight chamber" in Fig. 1 is always, in the more recent work, filled with one of these gases.

Furthermore, the plane wave of compression as it proceeds through the plate must be exceedingly regular. It is known that in such a compression the pressure rise

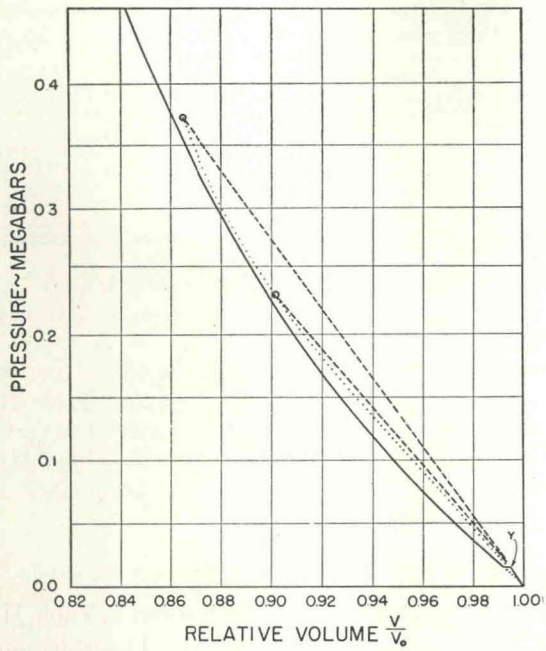

FIG. 2. Solid line represents a hypothetical isentropic equation of state of steel subjected to uniaxial compression, showing effect of shearing stress below the dynamic yield point Y. Shock wave velocity is given by the expression $D=(-\Delta p/\Delta V)^{\frac{1}{2}}V$, where V is the specific volume of the material ahead of the shock; Δp is the incremental shock pressure; and ΔV, the change in specific volume due to the shock.

Below the dynamic yield point, compressional waves are propagated with the sound velocity, proportional to the square root of the slope of the solid line.

Above dynamic yield point, compressional waves are propagated with velocities proportional to the square root of the slope of the dashed lines. By joining the end points of the dashed lines, one obtains the representation of the "Rankine-Hugoniot" equation of state shown by the dotted curve. This lies above the isentropic because of the entropy change under shock conditions. Upper dashed line represents a shock traveling with the velocity of sound at low pressures, i.e., a shock not preceded by an elastic wave.

is exceedingly abrupt and that the amplitude of this abrupt rise is constant provided the material behind the wave is uniformly compressed. Constancy of pressure behind the shock front may be achieved approximately by using a sufficiently large block of high explosive. If, however, the wave is not plane, its space configuration varies as it proceeds, and if the compression behind the front is not uniform, the magnitude of the virtually discontinuous pressure change is not constant. Furthermore, even if the wave is perfect in terms of the above criteria the plane of the waves may not be parallel to the plane of the plate. A variety of special precautions has been introduced to minimize effects of these possible sources of error. Continual improvement in the preparation of the H.E. and the "lens" have virtually eliminated departures from planeness and tilt in the wave itself. A small residual tilt of the front does not affect (to errors of the first order) the inferred velocities if the pins are properly arranged in small circles. In some experiments, as many as nine circles of eight pins each are used to supply simultaneous information.

In addition to the above difficulties it has been found that small irregularities or scratches in the surface of the plate result in jets which may cause erratic pin discharge. Indeed owing to the polycrystalline structure of the metal itself, some irregularities in the moving free surface are invariably present, the magnitude of these irregularities being of the order of the size of the individual metallic crystal grains. Because of this unavoidable roughness, it is not practical to make free surface velocity measurements over extremely short ranges of motion. Experiment has shown, however, that these irregularities are not too serious if the total range covered by the pins exceeds 5 mm.

A further limitation on the method results from the fact that in certain materials (e.g., steel) an elastic wave of compression moves with a higher velocity than the shock wave up to a certain pressure which depends on the dynamic yield point (see Fig. 2). In such cases, the necessary information can be achieved by the use of piezoelectric crystals (see Figs. 3 and 4).

One further technical experimental point deserves brief mention. As has been mentioned, some decay of pressure is encountered with increasing thickness of the plate. It is thus essential that the shock wave velocity and the mass velocity be obtained for an equivalent particle, namely a particle close to the free surface of the plate. But the probes for measuring propagation velocity are perforce distributed through the thickness of the plate, and, since the amplitude of the shock is varying, so also does the propagation velocity vary. The simplest way of finding shock velocity at the free surface is to make the portion of the plate where the shock velocity is measured somewhat thicker than the portion where the free surface velocity is measured, so that an average value for the former will be compatible with the observed value of the latter.

From the measured free surface velocity, the mass velocity of the compressed material may be inferred. It is, of course, necessary to complete the measurement of free surface velocity before reverberations can occur in the target plate. Otherwise, one obtains a measure not of mass velocity but of momentum transfer from explosive to plate. Furthermore because of the decay of pressure behind the shock front, one might expect the *observed* free surface velocity to diminish as the motion proceeds, but such an effect has not been detected. With these considerations in mind each set of 8 contactors is usually spaced over an interval of about 5 mm from the

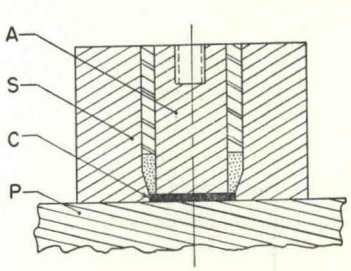

FIG. 3. Assembly for holding piezoelectric crystal in place. *A* Metal electrode and inertial support for crystal. *S* Guard ring. *C* Crystal. *P* Specimen through which shock-wave proceeds.

back surface of the plate, though for very thin plates an even closer spacing may be necessary in spite of the uncertainties caused by roughness of the moving free surface as previously mentioned, and the shorter reverberation time.

In order to obtain extensive data on the equation of state of the material under study, it is necessary to produce in the specimen compressional shock waves of arbitrary amplitude. There are three ways in which this has been accomplished:

(1) By increasing the thickness of the specimen, the pressure decays naturally, because the amplitude of the pressure discontinuity remains constant only if the pressure in the compressed material is everywhere uniform. With blocks of high explosive of finite dimensions, this condition is not satisfied, and a continuous degradation of shock pressure is always encountered.

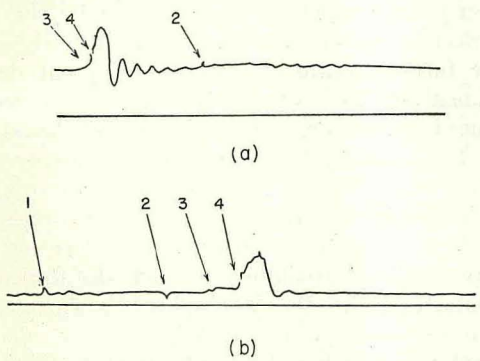

FIG. 4. Oscillograph records of shock-wave profile after moving through (a) 0.25 in. (b) 1.25 in. of SAE 4340 steel. (1) is cross talk from (a); (2) is the synchronizing time pip for the two records; (3) is the elastic-wave front; and (4) is the main shock front. Total sweep length is 10 μsec. Elastic pressure is 0.0157 mb; peak shock pressure in (b) is 0.225 mb; elastic velocity is 0.585 cm/μsec; and shock velocity is 0.510 cm/μsec. (Oscillations in shock front are reverberations in crystal probes.)

(2) The detonation pressure may be varied by using different compositions of high explosive.

(3) The amplitude of the transmitted shock may be adjusted by placing an intermediate metal between the block H.E. and the specimen "plate," as would be possible with acoustic waves.

OBSERVED DATA

The data obtained by the measurements made in 1945 are summarized in the first two columns of Table II. The more recent measurements made on Duralumin are listed in the first two columns of Table III.

In the foregoing two tables, the recorded data represent averages taken from a large number of individual determinations. In Table II the standard deviation for both wave velocities is of the order of 2%. Unfortunately, in the case of steel, the presence of the elastic wave renders the computation of shock pressure and compression somewhat uncertain; the magnitude of the correction appears, however, to be less than 1%. In Table III the standard deviations as computed from the residuals do not exceed 0.5% in any case. In compiling both tables the original oscillographic data were analyzed by the method of least squares. Table II is based on the assumption that the measured free surface velocity is twice the mass velocity. In Table III a correction to this approximation has been made. All units are as specified in Table I.

TABLE II. Early data on aluminum, cadmium, and steel.

Material	Shock velocity	Free surface velocity	$\eta-1$ (computed)	Pressure (computed)	Pressure (crystal)
Aluminum	0.738	0.295	0.250	0.294	——
Cadmium	0.396	0.145	0.224	0.248	0.231
Steel		0.115	0.122	0.223	(calibration)
Shock wave	0.510 (Average)	0.166	0.195	0.332	0.324
Elastic wave	0.588	0.00667	0.006	0.015	0.0157

NUMERICAL COMPUTATIONS AND RESULTS

In order to reduce the shock pressures in Table II to adiabatic pressures, the data of Table IV are required. The data of the first four columns were computed from material to be found in the usual sources, notably Birch's Handbook,[3] the Metals Handbook,[4] and the Handbook of Chemistry and Physics.[5]

The values of β_s in the fifth column are those deduced by correcting the data of Table II to isentropic conditions and then fitting Eq. (17) to the observed point.

In the case of steel, only the point obtained at the lower shock pressure was used in computing the value of β_s. At the higher pressure, the computed value of μ appears to be much too large, and would imply an even smaller value of β_s. Further work will be required to verify the discrepancy between Bridgman's work and ours, but the available data seem worth recording because of the interest which may attach to the peak pressure as recorded by the tourmaline crystal and the relatively good agreement between this pressure and that computed by Eq. (2).

Values of β_s may also be computed by an analysis of Bridgman's more recent work,[2] and for aluminum and iron these appear in the last column of Table IV.

TABLE III. Recent data on 24 ST Duralumin.

Shock velocity	Free surface velocity	Mass velocity (computed)	$\eta-1$ (computed)	Shock pressure (computed)	Isentropic pressure (computed)
0.6460	0.1629	0.0814	0.1442	0.1462	0.1435
0.6850	0.2254	0.1126	0.1967	0.2144	0.2079
0.7005	0.2395	0.1196	0.2059	0.2329	0.2250
0.7426	0.3014	0.1503	0.2538	0.3103	0.2952
0.7520	0.3179	0.1584	0.2668	0.3312	0.3139

[3] Francis Birch, *Handbook of Physical Constants* (Geological Society of America, 1942).

[4] *Americal Society of Metals*, "Metals handbook," 1948.

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TABLE IV. Thermodynamic properties of selected elements.

Element	ρ_0	α_s	C_v	$(\partial p/\partial T)_v$	β_s	β_s (Bridgman)
Aluminum	2.699	0.742	0.108	0.56	1.53	1.69±0.04
Cadmium	8.65	0.48	0.014	0.46	2.20	···
Steel	7.84	1.69	0.054	0.46	0.8	2.4 ±0.1

The standard deviations for these values are simply indications of the precision with which the empirical equation of state fits the observed points and should not be taken as an indication of the experimental accuracy. The relatively better precision shown in the case of the aluminum is owing chiefly to the fact that the aluminum is much more compressible, so that a greater range of values of the compression are available for analysis. The correction from β_T to β_s amounts to $+0.01$ for aluminum when computed by Eq. (22); it has not been computed for steel.

The corrections required to reduce the data of Table III are shown graphically in Fig. 4. The plotted values of $(\sigma-u)/u$ are subject to considerable uncertainty, but the values shown for p_D-p_s should be accurate to 5% or better. In analyzing the reduced data, a value $\alpha_s=0.760$ was arbitrarily selected. This value was inferred from measurements of the velocity of sound in Duralumin. The method of least squares was then used to infer the value $\beta_s=1.57\pm0.02$ for this material. Correction for the presence of copper [see Eqs. (25) and (26)] gives corresponding values for pure aluminum of $\alpha_s=0.753$ and $\beta_s=1.53$. Correction of these to isothermal conditions [see Eqs. (21) and (22)] yields $\alpha_T=0.726$, $\beta_T=1.52$. Corresponding coefficients computed from Bridgman's data again by the method of least squares are $\alpha_T=0.715\pm0.001$, $\beta_T=1.68\pm0.04$. The discrepancy between the two values of α is of no immediate concern, since the shock wave measurements are not used for this determination. However, part of the discrepancy between the two values of β_T results from the fact that the α's do not agree. If our arbitrary choice of α is made in such a way as to give agreement with the figure deduced from Bridgman's work, then the discrepancy between the β's drops from 0.16 to 0.11. One might reasonably inquire as to whether a three-term equation of state would better fit our data, and yield better agreement between our results and Bridgman's. Unfortunately if the second-order coefficients were made to agree, the third-order coefficient as deduced from our data would be negative, and hence could hardly be useful for purposes of extrapolation.

It should of course be noted that Bridgman's measurements extend to only 30 000 kg/cm², whereas ours cover roughly ten times this range. Since Eq. (17) is purely empirical, it is to be expected that different sets of coefficients would be required to give the best representations of the data over the two different ranges.

CONCLUSIONS

Determination of shock and mass velocities in material subject to explosive stress yields data for equations of state up to 0.3 megabar or more. With care the precision attainable is about 0.5% on the velocity measurements, but because of the extreme pressures that may be reached, even this precision provides a more precise determination of the second-order term in the equation of state than is possible with static measurements. In the case of aluminum, part or all of the discrepancy between the coefficients determined statically and those determined dynamically may be owing to the different pressure ranges to which the respective measurements apply, though the sign of the discrepancy suggests that this explanation is inadequate.

In Fig. 5 there has been included an estimate of the final temperature increase in the Duralumin after the expansion to atmospheric pressure. [This estimate is based on the entropy change as calculated by Eq. (16), and has been converted to centigrade degrees.] It will be observed that the melting point of the aluminum would be reached by a 0.6-megabar shock, and it seems likely that the measurements would become impracticable under these conditions. In any event the problem of correcting free surface velocity to obtain mass velocity would become much more complicated.

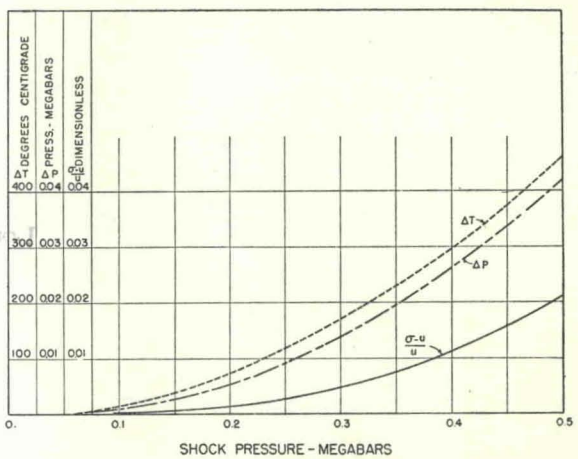

FIG. 5. Excess of shock pressure over adiabatic pressure (ΔP), temperature increase after expansion (ΔT), and fractional excess free surface velocity $(\sigma-u)/u$ as functions of pressure.

A final word about precision seems in order. It might be imagined on casual examination of Eqs. (1) and (2) that D and u must be determined with equal precision in order to obtain useful data on the equation of state. Fortunately, however, u need not be known with as great precision as D. To show this, let us consider the problem of determining β in terms of u and D by means of Eqs. (1), (2), and (17), assuming α to be known. The fractional error $\Delta\beta/\beta$ to be feared in β due to fractional

errors $\Delta u/u$ and $\Delta D/D$ in u and D, respectively, is

$$(\Delta\beta/\beta)=(u/\beta)(\partial\beta/\partial u)(\Delta u/u) + (D/\beta)(\partial\beta/\partial D)(\Delta D/D). \quad (27)$$

Calculation reveals that for the Duralumin, at the highest pressure attained, $(u/\beta)(\partial\beta/\partial u)=-1.9$ and $(D/\beta)(\partial\beta/\partial D)=7.8$ so that four times the precision is required on D to make the errors of equal magnitude. Accuracy of 0.5% on D would imply 4% accuracy on β. In view of the fact that five points were used for our final determination of β, the calculated standard deviation of a little less than 2% appears to be compatible with these considerations.

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